

2. Homogenization of Open and Closed Porosity in Metallic Fuel

2.1 Introduction

Modern metallic fuel designs typically have smear densities (defined as the cross-sectional area of the fuel divided by the area inside the cladding) of about 75% or less. As the fuel swells within the confines of the cladding, it becomes sufficiently porous so that some of the porosity becomes interconnected and allows fission gas to be released into the plenum. The result is reduced gas pressure in the interconnected pores, smaller loads applied to the cladding, and consequently extended lifetime of the fuel system. Although much of the fission gas escapes, a significant amount is typically retained in isolated pores. The fuel may therefore be viewed locally as a composite comprised of regions containing open, interconnected porosity and regions containing closed, isolated porosity.

The treatment of internally pressurized porous materials whose fully dense matrix undergoes inelastic deformation is well established [1, 2, 3, 4, 5], and depends on two key ingredients: (1) the effect of internal pore pressure can be accounted for by adding the pressure to the externally applied hydrostatic stress; and (2) the behavior of the porous material itself can be represented using an appropriate constitutive law [6, 7, 8]. Though well-documented, with the exception of reference [5] we have not seen an argument put forth as to why the first assumption is valid. We therefore provide a brief argument for it here.

The larger problem, however, is how to account for the regions of open and closed porosity within a representative material volume, with each region having a distinct pore pressure. The objective of the present work is to develop a second homogenization step that accounts for the different pore pressures within a representative volume of fuel. This second stage of homogenization, built on well-established models used to describe each porous region individually, provides a basis for development of a physically consistent metallic fuel performance model. Before developing this second-stage homogenization, however, we first provide an overview of how it fits into an overall fuel model and a description of the first stage of homogenization.

2.2 Outline of a Fuel Model

A practical fuel model requires a simple relationship between the evolving pore structure and fission gas release. We assume that local fission gas release is a monotonically increasing function of the average pore volume fraction [9] and that the pore volume fractions within the open- and closed-porosity regions are equal. The first assumption is rooted in percolation theory while the second is an assumption of convenience. If f_C and f_O are the volume fractions of the closed and open regions and v_f^C and v_f^O are their pore volume fractions, we assume that $v_f = v_f^C = v_f^O$. Since the regional porosities are equal, the fraction of each region occupied by fuel alloy is also equal. At a given burnup, the amount of fission gas generated within a region is directly proportional to the amount of fuel alloy it contains. The fractional gas release can therefore be related directly to the fraction of the composite occupied by open porosity regions

$$f_O = \text{FGR}(v_f) \quad f_C = 1 - \text{FGR}(v_f). \quad (2.1)$$

In addition, assuming a uniform pore volume fraction results in significantly simpler expressions for tracking the evolution of fuel microstructure since pore volume fraction becomes the sole state variable. Finally, as will be shown subsequently, equal regional porosity leads to a particularly simple and effective mechanical homogenization.

The adoption of a fission gas release model based on fuel porosity does, however, introduce an additional difficulty. Once the fuel has filled the confines of the cladding and is constrained, the volumetric swelling rate diminishes substantially. If all solid fission products are assumed to accumulate within the fuel alloy, the porosity in the fuel and consequently the fission gas release will begin to decline, in contrast to fission gas release data that show a reduced but still significant rate of increase with burnup after fuel/cladding contact [10].

This discrepancy can potentially be resolved if the solid fission products are partitioned according to whether they remain in the fuel alloy or precipitate within existing pores. Figure 2.1 shows that many of the precipitated solid fission products appear to be separated from the surrounding fuel alloy over much of their perimeter rather than forming a coherent interface with the alloy matrix, suggesting that the precipitates themselves will not significantly affect the percolation behavior of the pores. Motivated by the behavior observed during historical melt refining of irradiated metallic fuel, we hypothesize that only the fissium elements (Zr, Nb, Mo, Tc, Ru, Rh, and Pd; hereafter denoted as Fs) remain within the fuel alloy, while the other solid fission products precipitate within the pores.

To assess whether this assumption can provide a physically plausible post-contact model of porosity evolution, we need to estimate the swelling rates associated with the accumulation of fissium and non-fissium solid fission products, as well as the reduction in alloy volume associated with the loss of uranium atoms through fission. In oxide fuels, such estimates are complicated by the large effect that the chemical state of the solid fission products can have on their effective atomic volumes. For example, Olander estimates solid fission product swelling rates ranging from approximately 0.15 to 0.45% per atom percent burnup depending on relatively small variations in the initial stoichiometry of uranium dioxide [13].

In metallic fuels, reasonable estimates of solid fission product swelling can be made considerably simpler because the formation of metallic and intermetallic phases generally has a much smaller effect on the effective atomic volume. In his work on intermetallic compounds, Pauling [14] classified metals as hypoelectronic, buffer, or hyperelectronic according to the balance between valence electrons and available metallic bonding orbitals, and showed that electron transfer between unlike classes can increase bonding valence and produce contraction relative to elemental volume additivity. Metallic fuel chemistry is dominated by hypoelectronic–hypoelectronic and hypoelectronic–buffer combinations; in the examples considered by Pauling, departures from elemental volume additivity for such combinations are typically only a few percent. As a direct check of this approximation, unit cell volumes were calculated from measured lattice constants for UZr_2 [15] and U_2Mo [16] and were found to differ from the sums of the corresponding elemental atomic volumes by only about 1–2%. Elemental atomic volumes can therefore be used to obtain reasonable estimates of the volume changes associated with

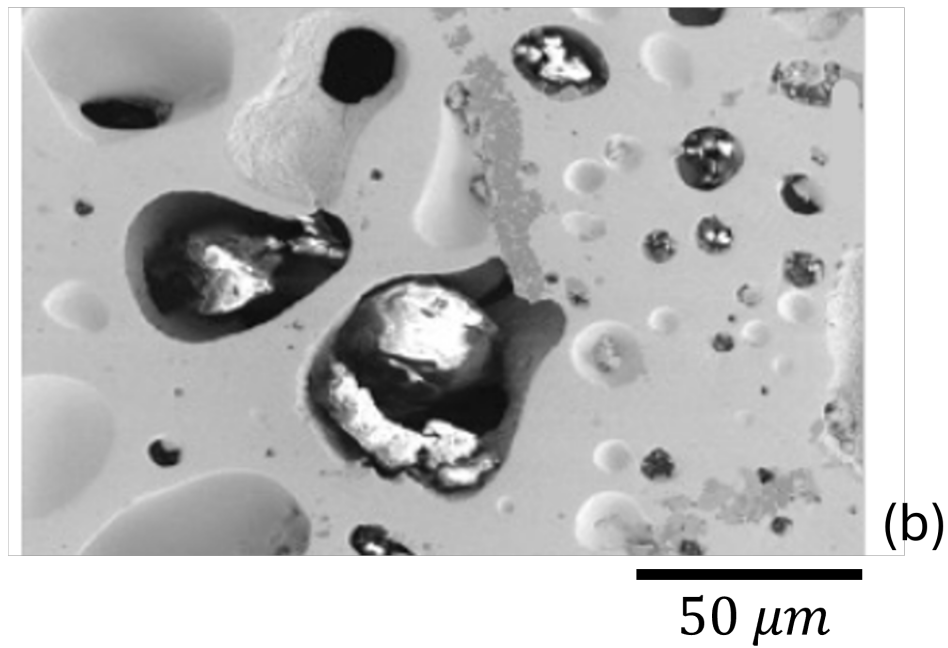
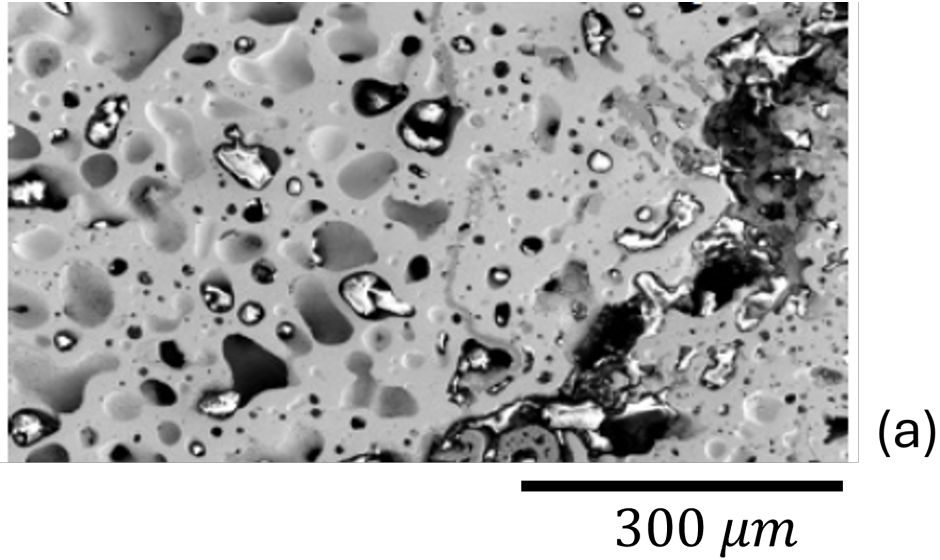


Figure 2.1: Metallography showing solid fission products that have precipitated in pores within the fuel alloy. Adapted from references [11, 12], annotations added.

the accumulation of solid fission products in metallic fuel.

Based on these observations, solid fission product swelling rates can be estimated from fast-neutron fission yields [13] and elemental molar or atomic volumes [17]. Where several elements are combined into a single fission-yield group, we use the arithmetic mean molar volume of the elements in that group. The fission contribution is assigned to the fuel alloy, from which the atomic volume of one uranium atom consumed per fission is subtracted to obtain the net alloy swelling rate. The remaining non-gaseous, non-fission products provide the

precipitate swelling rate, and the sum of the alloy and precipitate contributions gives the total solid fission product swelling rate. The resulting values, summarized in Table 2.1, allow us to assess whether the proposed partition produces a physically plausible post-contact evolution of porosity and they also provide an independent estimate of solid fission product swelling parameters for use in fuel performance models.

Table 2.1: Estimated solid fission product swelling rates in metallic fuel.

Quantity	Swelling Rate (cm ³ /fission)
Fuel Alloy	-0.720×10^{-23}
Non-fissium precipitates	$+5.233 \times 10^{-23}$
Net swelling rate	$+4.513 \times 10^{-23}$

For the continuum model, this partition requires a distinction between the gross geometrical pore volume, which controls pore interconnection, and the portion of that volume actually available to fission gas. In the following, all volume fractions defined here are with respect to the total local volume of the *fuel alloy+Fs / fission gas / non-Fs solid fission products* composite.

$$v_f^{\text{gas}} + v_f^{\text{Non-Fs}} = v_f, \quad (2.2)$$

where v_f^{gas} is the fraction of the total local volume available to fission gas and $v_f^{\text{Non-Fs}}$ is the fraction occupied by precipitated non-fissium solid fission products. Given the pore volume fraction v_f , the volume fraction occupied by the fuel alloy, including retained fissium and accounting for uranium consumed by fission, is simply

$$v_f^{\text{alloy}} = 1 - v_f. \quad (2.3)$$

Thus, the precipitates reduce the free volume available for fission gas but, because they do not form a coherent, hermetic interface with the fuel alloy matrix, the total pore volume fraction is used when estimating the degree of pore interconnection and amount of fission gas released. These same volume fractions will also be useful in estimating the effective thermal conductivity of the fuel, although treatment of that issue is beyond the scope of this document.

With these volume fraction definitions, the effect of the proposed fission product partition on porosity evolution can now be seen directly. The pore volume fraction evolves according to

$$\dot{v}_f = (1 - v_f) \left(\dot{\epsilon}_V - \dot{\epsilon}_V^{\text{alloy}} \right), \quad (2.4)$$

where $\dot{\epsilon}_V$ is the macroscopic volumetric strain rate of the porous fuel and $\dot{\epsilon}_V^{\text{alloy}}$ is the intrinsic volumetric strain rate of the fuel alloy. After fuel/cladding contact is initiated, $\dot{\epsilon}_V$ is approximately zero. The negative alloy swelling rate provided in Table 2.1 therefore allows v_f —and hence a monotonically increasing $\text{FGR}(v_f)$ —to continue increasing even after macroscopic swelling of the fuel has been effectively constrained by the cladding.

In the complete fuel model, the macroscopic volumetric strain rate $\dot{\epsilon}_V$ appearing in Equation 2.4 will be obtained after the second stage of homogenization. Before addressing the second stage procedure, we will first examine a fundamental assumption used in the first stage, which is that the effect of an internally pressurized pore can be accounted for by adding that pressure to the externally applied hydrostatic stress.

2.3 Stage 1 Homogenization: Internally Pressurized Porous Materials

The first stage of homogenization replaces the explicit pore structure within each open- or closed-porosity region with a homogeneous porous material. Before choosing a particular constitutive model for that material, however, we first consider how the internal pore pressure enters its mechanical response. Under the assumptions of negligible elastic strain rates and incompressible, pressure-insensitive inelastic deformation of the fully dense matrix, the effect of a spatially uniform internal pore pressure can be accounted for exactly by adding that pressure to the externally applied hydrostatic stress.

Consider an internally pressurized pore within a fully dense matrix, and let $\sigma^m(\mathbf{x})$ denote the actual Cauchy stress in the matrix. With tension taken as positive, the traction acting on the pore surface is

$$\sigma^m \mathbf{n} = -P\mathbf{n}, \quad (2.5)$$

where P is the internal pore pressure and $\mathbf{n}$ is the outward normal from the matrix. Define the transformed stress field

$$\sigma^{m*}(\mathbf{x}) = \sigma^m(\mathbf{x}) + P\mathbf{I}, \quad (2.6)$$

where $\mathbf{I}$ is the identity tensor. Boldface quantities are used to denote vectors or tensors.

For a spatially uniform pore pressure, the transformed stress field satisfies the same local equilibrium equations as the actual stress field because

$$\nabla \cdot \sigma^{m*} = \nabla \cdot \sigma^m + \nabla P = \nabla \cdot \sigma^m. \quad (2.7)$$

At the pore surface,

$$\sigma^{m*} \mathbf{n} = \sigma^m \mathbf{n} + P\mathbf{n} = \mathbf{0}. \quad (2.8)$$

Thus, the transformed problem is equivalent to one containing a traction-free pore, with the externally applied stress state increased by the hydrostatic stress $P\mathbf{I}$.

The second part of the argument concerns deformation of the fully dense matrix. Adding a hydrostatic stress does not alter the stress deviator:

$$\text{dev } \sigma^{m*} = \text{dev } (\sigma^m + P\mathbf{I}) = \text{dev } \sigma^m. \quad (2.9)$$

Consequently, if the inelastic response of the fully dense matrix is incompressible and pressure-insensitive, the actual and transformed stress fields produce exactly the same local inelastic strain-rate field. With elastic strain rates neglected, they therefore produce the same deformation of the porous material.

A formal homogenization treatment of this hydrostatic stress transformation was given by Vincent et al. [5]; here the direct argument makes clear why it applies to the present formulation. The result is not specific to any particular porous constitutive model or pore geometry. Any physically based porous-material model derived from an incompressible, pressure-insensitive fully dense matrix may incorporate a uniform internal pore pressure by adding that pressure to the externally applied hydrostatic stress. Under the assumptions stated above, this treatment is exact rather than an approximation.

Having established how internal pore pressure enters the first-stage homogenization, the remaining issue is the choice of a constitutive model for the porous material. Several physically based models are available. The Gurson model [6] describes rate-independent plastic flow, whereas the models of Duva and Crow [7] and Sofronis and McMeeking [8] describe porous materials whose fully dense matrix undergoes rate-dependent power-law deformation. Because the flow behavior of metallic fuel is expected to be strongly strain-rate dependent, corresponding to a relatively low stress exponent, a rate-dependent formulation is more appropriate for the present application. We therefore use the Duva–Crow model in the present work; the closely related Sofronis–McMeeking model may be considered in future studies.

At a specified temperature, the rate-dependent deformation of the fully dense fuel alloy can be characterized by a simple uniaxial relation,

$$\dot{\epsilon} = \dot{\epsilon}_0 \left(\frac{\sigma}{\sigma_0} \right)^n, \quad (2.10)$$

where $\dot{\epsilon}_0$, σ_0 , and n characterize the uniaxial response. The same relation can be expressed in terms of the flow potential

$$\Psi = \frac{\dot{\epsilon}_0 \sigma_0}{n+1} \left(\frac{\sigma}{\sigma_0} \right)^{n+1}, \quad (2.11)$$

for which $\dot{\epsilon} = \partial \Psi / \partial \sigma$.

The familiar von Mises construction provides an elegant means of extending this simple uniaxial characterization to a general multiaxial stress state for a fully dense, isotropic material whose inelastic deformation is pressure-insensitive. For the homogenized material, define

$$\sigma_H = \frac{1}{3} \text{tr } \boldsymbol{\sigma}, \quad (2.12)$$

$$\mathbf{S} = \boldsymbol{\sigma} - \sigma_H \mathbf{I}, \quad (2.13)$$

$$\bar{\sigma} = \left(\frac{3}{2} \mathbf{S} : \mathbf{S} \right)^{1/2}. \quad (2.14)$$

Replacing the uniaxial stress σ in Equation 2.11 by the von Mises stress $\bar{\sigma}$ gives

$$\Psi_{\text{dense}} = \frac{\dot{\epsilon}_0 \sigma_0}{n+1} \left(\frac{\bar{\sigma}}{\sigma_0} \right)^{n+1}, \quad (2.15)$$

thereby extending material behavior characterized by a uniaxial test to a general three-dimensional stress state.

The same idea can be extended to a porous solid, but the von Mises stress alone is no longer sufficient. Although the fully dense matrix deforms incompressibly, hydrostatic stress can produce volumetric deformation of the porous material through growth or collapse of its pores. Duva and Crow account for this behavior by introducing the effective stress

$$\sigma_{\text{eff}} = (a\bar{\sigma}^2 + b\sigma_H^2)^{1/2}, \quad (2.16)$$

where the coefficients a and b depend on pore volume fraction and stress exponent. The corresponding flow potential is

$$\Psi_{\text{DC}} = \frac{\dot{\epsilon}_0 \sigma_0}{n+1} \left(\frac{\sigma_{\text{eff}}}{\sigma_0} \right)^{n+1}, \quad (2.17)$$

and differentiation with respect to the stress tensor gives the associated flow rule

$$\dot{\epsilon} = \frac{\dot{\epsilon}_0}{\sigma_0} \left(\frac{\sigma_{\text{eff}}}{\sigma_0} \right)^{n-1} \left[\frac{3a}{2} \mathbf{S} + \frac{b\sigma_H}{3} \mathbf{I} \right]. \quad (2.18)$$

The porosity dependence is contained in

$$a(n, v_f) = \frac{1 + \frac{2}{3}v_f}{(1 - v_f)^{2n/(n+1)}}, \quad (2.19)$$

and

$$b(n, v_f) = \frac{3}{4} \left[\frac{n^{-1}v_f^{1/n}}{1 - v_f^{1/n}} \right]^{2n/(n+1)}. \quad (2.20)$$

Much like the von Mises stress, the Duva–Crow formulation retains the same uniaxial material parameters $\dot{\epsilon}_0$, σ_0 , and n used to characterize the fully dense alloy, while extending that characterization to the more complicated deformation of a porous solid under multiaxial stress states. In the limit $v_f \rightarrow 0$, $a \rightarrow 1$ and $b \rightarrow 0$, so that $\sigma_{\text{eff}} \rightarrow \bar{\sigma}$ and the Duva–Crow formulation reduces to the conventional von Mises power-law description of the fully dense material.

Equations 2.16–2.20 are written for a porous material with traction-free pores. For an internally pressurized region, the result derived above allows the same constitutive model to be used by replacing the hydrostatic stress with

$$\sigma_H^* = \sigma_H + P, \quad (2.21)$$

while leaving the stress deviator unchanged. Thus, once v_f , P , and the externally applied stress are specified, the first-stage homogenization provides the strain-rate response of either an open- or closed-porosity region. The remaining problem is to combine the responses of these regions, which generally have different pore pressures, to obtain the macroscopic response of the composite fuel.

2.4 Stage 2 Homogenization: Two Regions with Different Internal Pressures

The second-stage homogenization seeks a simple constitutive model that reproduces the macroscopic response of the face-centered cubic (FCC) unit cell shown in Figure 2.2(a), for use in full-scale fuel-performance calculations. In the FCC finite element model, spherical closed-porosity regions are explicitly resolved within an open-porosity matrix, while the pore structure within each region is represented by the Stage 1 Duva–Crow constitutive model. In this study, the FCC finite element solution is taken as the reference behavior, or “truth,” that the simpler Stage 2 models are intended to reproduce.

Let V be the volume of the FCC unit cell, with $V_C = f_C V$ and $V_O = f_O V$ the volumes occupied by the closed- and open-porosity regions. Let $\sigma(\mathbf{x})$ and $\dot{\epsilon}(\mathbf{x})$ denote the local, position-dependent stress and strain-rate fields within the unit cell. We use $\langle x \rangle$ to denote the volume-averaged value of a quantity x over the complete unit cell, and $\langle x \rangle_C$ and $\langle x \rangle_O$ for averages over the two regions:

$$\langle x \rangle = \frac{1}{V} \int_V x dV, \quad \langle x \rangle_r = \frac{1}{V_r} \int_{V_r} x dV, \quad r = C, O. \quad (2.22)$$

The macroscopic stress and strain-rate tensors are therefore

$$\Sigma = \langle \sigma \rangle, \quad \dot{\mathbf{E}} = \langle \dot{\epsilon} \rangle. \quad (2.23)$$

The components of Σ are the externally applied macroscopic stresses acting on the unit cell.

The regional average stresses and strain rates are defined directly from the FCC fields as

$$\sigma_C = \langle \sigma \rangle_C, \quad \sigma_O = \langle \sigma \rangle_O, \quad \dot{\epsilon}_C = \langle \dot{\epsilon} \rangle_C, \quad \dot{\epsilon}_O = \langle \dot{\epsilon} \rangle_O. \quad (2.24)$$

Consequently, the macroscopic averages satisfy the exact identities

$$\Sigma = f_C \sigma_C + f_O \sigma_O, \quad \dot{\mathbf{E}} = f_C \dot{\epsilon}_C + f_O \dot{\epsilon}_O. \quad (2.25)$$

These relations require no homogenization assumption. The candidate Stage 2 models approximate the regional averages by treating each region as homogeneous and imposing one of two simple limiting relations. A uniform strain, or Voigt-type, construction assumes

$$\dot{\epsilon}_C = \dot{\epsilon}_O = \dot{\mathbf{E}}, \quad (2.26)$$

with the regional stresses determined from the Stage 1 constitutive model subject to the exact stress-average relation in Equation (2.25). A uniform externally applied stress, or Reuss-type, construction instead assumes

$$\sigma_C = \sigma_O = \Sigma, \quad (2.27)$$

and obtains the macroscopic strain rate by averaging the two Stage 1 regional strain-rate responses according to Equation (2.25). The qualification *externally applied* is important: the same macroscopic stress is assigned to both regions, but their different internal pressures, P_C and P_O , produce different pressure-adjusted constitutive driving stresses and therefore

different strain rates. The uniform strain and uniform externally applied stress assumptions are expected to provide upper- and lower-bound estimates, respectively, for the flow resistance of the two-region composite.

There was no a priori reason to expect either of these simple constructions to reproduce the FCC response particularly well. We therefore evaluated both against the FCC reference solution for all combinations of the parameters listed in Table 2.2, giving a total of 96 cases. The externally applied macroscopic stress was specified by its Mises stress $\bar{\Sigma}$ and hydrostatic stress $\Sigma_H = \text{tr } \Sigma/3$, with the Mises stress applied in the 2-direction:

$$\Sigma = \begin{bmatrix} \Sigma_H - \bar{\Sigma}/3 & 0 & 0 \\ 0 & \Sigma_H + 2\bar{\Sigma}/3 & 0 \\ 0 & 0 & \Sigma_H - \bar{\Sigma}/3 \end{bmatrix}. \quad (2.28)$$

Table 2.2: Parameters used in the 96-case FCC homogenization study.

Quantity	Value(s)
Reference stress, σ_0	100 MPa
Reference creep rate, $\dot{\epsilon}_0$	$1.0 \times 10^{-3} \text{ s}^{-1}$
Open-region volume fraction, f_O	0.60, 0.80
Regional pore volume fraction, $v_f^C = v_f^O = v_f$	0.25, 0.40
Closed-pore pressure, P_C/σ_0	1.00
Open-pore pressure, P_O/σ_0	0.10, 0.50
Applied Mises stress, $\bar{\Sigma}/\sigma_0$	0.10, 0.50
Applied hydrostatic stress, Σ_H/σ_0	-0.10, -0.75
Stress exponent, n	1, 3, 5

Figure 2.3 compares the effective deviatoric and volumetric strain rates predicted by the two homogenized models with the FCC results. The uniform strain approximation can differ substantially from the FCC calculations, but surprisingly the uniform externally applied stress model reproduces them quite accurately over the range examined.

The question then arises as to why such a simple Reuss-type model should work so well. Inspection of the finite element results for the unit cell provides a clue. Figure 2.2(b) shows the local hydrostatic stress $\sigma_H(\mathbf{x}) = \text{tr } \boldsymbol{\sigma}(\mathbf{x})/3$ for a representative nonlinear case. Although the closed- and open-pore pressures differ by $0.5\sigma_0$, the local hydrostatic stress differs from the applied macroscopic value Σ_H by less than about 0.2% throughout the unit cell. This near uniformity helps explain the accuracy of the uniform externally applied stress approximation in this case. As shown next, however, the linear creep limit reveals additional surprising results.

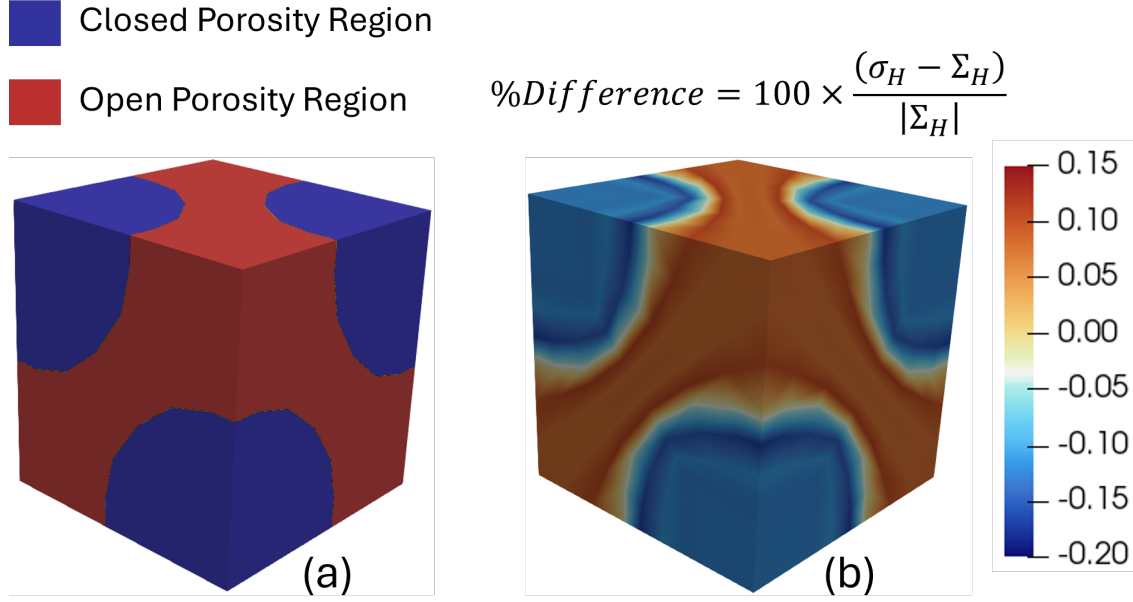


Figure 2.2: (a) One-eighth face-centered cubic (FCC) unit cell containing spherical closed-porosity regions within an open-porosity matrix; (b) percentage difference $100(\sigma_H - \Sigma_H)/|\Sigma_H|$ for $n = 3$, $\bar{\Sigma} = 0.10\sigma_0$, $\Sigma_H = -0.75\sigma_0$, $P_C = \sigma_0$, $P_O = 0.5\sigma_0$, $v_f = 0.40$, and $f_O = 0.60$.

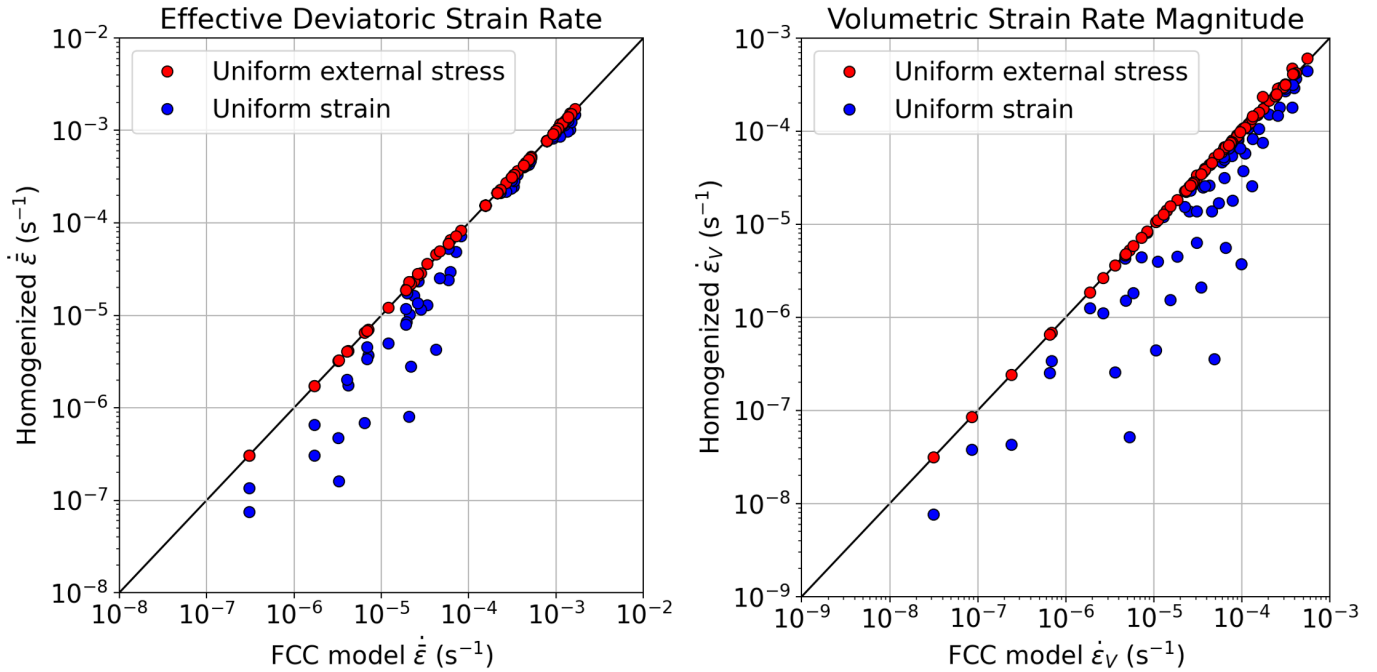


Figure 2.3: One-to-one comparisons of effective deviatoric and volumetric strain rates predicted by the uniform strain and uniform externally applied stress models with those obtained from the FCC unit-cell calculations.

2.5 Exact Homogenization for Linear Creep and Equal Porosity

For the limiting case of linear matrix creep with equal porosity in the two regions, the Duva–Crow flow rule is linear in the pressure-adjusted stress. At each point in the closed porosity/open porosity composite, we can write the creep strain rates as

$$\dot{\epsilon}(\mathbf{x}) = \frac{\dot{\epsilon}_0}{\sigma_0} \left[\frac{3a(v_f)}{2} \mathbf{S}(\mathbf{x}) + \frac{b(v_f)}{3} (\sigma_H(\mathbf{x}) + P(\mathbf{x})) \mathbf{I} \right], \quad (2.29)$$

where $P(\mathbf{x})$ is P_C or P_O . Because the porosity is the same in both regions, $a(v_f)$ and $b(v_f)$ are spatially uniform and can be taken outside the volume average. Using the definitions in Equation (2.23),

$$\dot{\mathbf{E}} = \frac{\dot{\epsilon}_0}{\sigma_0} \left[\frac{3a(v_f)}{2} \langle \mathbf{S} \rangle + \frac{b(v_f)}{3} (\langle \sigma_H \rangle + \langle P \rangle) \mathbf{I} \right]. \quad (2.30)$$

Since

$$\langle \mathbf{S} \rangle = \text{dev } \Sigma, \quad \langle \sigma_H \rangle = \Sigma_H, \quad \langle P \rangle = f_C P_C + f_O P_O, \quad (2.31)$$

the exact macroscopic strain rate is

$$\dot{\mathbf{E}} = \frac{\dot{\epsilon}_0}{\sigma_0} \left[\frac{3a(v_f)}{2} \text{dev } \Sigma + \frac{b(v_f)}{3} (\Sigma_H + f_C P_C + f_O P_O) \mathbf{I} \right]. \quad (2.32)$$

This is exactly the result obtained from the uniform externally applied stress construction: setting $\sigma_C = \sigma_O = \Sigma$ in the two Stage 1 regional models and volume averaging their strain rates gives Equation (2.32). Thus, for $n = 1$ and $v_f^C = v_f^O$, the Reuss-type construction gives the exact macroscopic strain-rate tensor.

More surprisingly, no information describing the arrangement, shape, or connectivity of the open- and closed-porosity regions appears in Equation (2.32). The result is therefore independent of cell morphology: the local stress field need not be uniform, because its detailed spatial distribution disappears upon volume averaging. Equivalently, the composite responds macroscopically as a single Duva–Crow material with porosity v_f and an effective pore pressure $f_C P_C + f_O P_O$. As an additional check, an orthorhombic arrangement of the open- and closed-porosity regions was analyzed rather than the FCC cell; the same macroscopic response was obtained, confirming numerically that the FCC agreement was not a consequence of cubic symmetry.

2.6 Nonlinear Creep

For nonlinear creep, $n > 1$, the factor $(\sigma_{\text{eff}}/\sigma_0)^{n-1}$ in Equation (2.18) varies spatially with the local stress state and cannot be taken outside the volume average. The exact homogenization derived for $n = 1$ therefore does not apply. The $n = 3$ and $n = 5$ cases from the 96-case study summarized in Table 2.2 were used to assess the uniform externally applied stress approximation for nonlinear creep. The median absolute errors relative to the FCC unit-cell results were 0.74% and 0.90% for the effective deviatoric and volumetric strain rates, respectively. The errors generally increased with stress exponent and porosity, and with greater pore-pressure contrast and stronger deviatoric loading. Thus, although the exact morphology-independent

result does not extend to $n > 1$, the uniform externally applied stress construction remained accurate for most of the nonlinear cases examined.

2.7 Future Work

The equal-porosity homogenization developed here will be implemented in HeavyMetal as part of a metallic fuel performance model. The implementation will combine the evolving porosity and open- and closed-region fractions described in Section 2.2 with distinct regional pore pressures and the Stage 2 homogenization developed above.

A subsequent extension will allow $v_f^C \neq v_f^O$. In that case the Duva–Crow coefficients a and b differ between the two regions and can no longer be taken outside the volume average, so the morphology-independent result of Section 2.5 no longer applies even for $n = 1$. A concentric composite-sphere solution with unequal regional porosities provides a reference problem for developing and evaluating the more general Stage 2 homogenization.

The metallography in Figure 2.1 also suggests a distinction in characteristic pore size between the two regions. Isolated pores tend to be smaller and more nearly spherical, whereas interconnected pores show evidence of growth and coalescence and are generally larger and more irregular. A more complete model could account for these differences and include the corresponding surface-energy contribution to pore pressure.

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